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STATISTICAL ANALYSIS OF AVERAGE, HIGH AND LOW WATERS OF THE TOPLICA RIVER

Abstract: The river Toplica springs on the east slopes of Kopaonik. It is the biggest left tributary of the Južna Morava, in which it empties at the town of Korvingrad. It is 130 km long and the surface of its river-basin amounts to 2180 km². This study estimates the probability of average, minimum and maximum discharge occurrence on the hydrological profile Pepeljevac on the Toplica River in the period 1951-2014. Pearson type III distribution was used for quantifying average, minimum and maximum annual discharge. Results of the study point out to significant fluctuations in the river Toplica discharge upstream the hydrological profile Pepeljevac, which is the base for further study and improvement water management planning in the basin. On the basis of probability of average annual discharges occurrence, a classification of years by water richness was done. Mann-Kendal test examined the trend of Toplica discharge, while Pettit, SNTH, Buishand and von Neumann tests analyzed homogeneity of the data on the observed profile. Analysis of average annual discharges shows that years moderately rich in water are the most numerous (29) with somewhat higher participation of rich in water (16) than dry (14) years. Coefficients of variation of maximum and minimum annual discharges for the Toplica river point out to significant fluctuations upstream the hydrological profile Pepeljevac. These results are the base for further study and improvement of water management planning in the basin.

Key words: The Toplica River, hydrological prognosis, classification of years by water richness

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Introduction

The Toplica is the biggest left tributary of Južna Morava, in which it empties at the town of Korvingrad. The Toplica emerges on the east slopes of Kopaonik by joining of the rivers of Djerekaruša and Lukovska. It is 130 km long and the surface of its river-basin amounts to 2180 km². Up to the town of Kuršumljija, the Toplica runs through a narrow and deep valley. From Kuršumljija, the valley is broader and shallower. Upstream from Kuršumljija the Toplica is 10-12 m broad at most and up to 0.90 m deep. Between Kuršumljija and Prokuplje it is up to 20 m broad and up to 1.50 m deep, while it is up to 25 m broad and 1-2 m deep downstream the course through Dobrič (Gavrilović, 2014). The biggest part of the Toplica discharge (75,76 %) is formed in the area from Kuršumljija to the village of Beloljin (Stanojević, 2001).

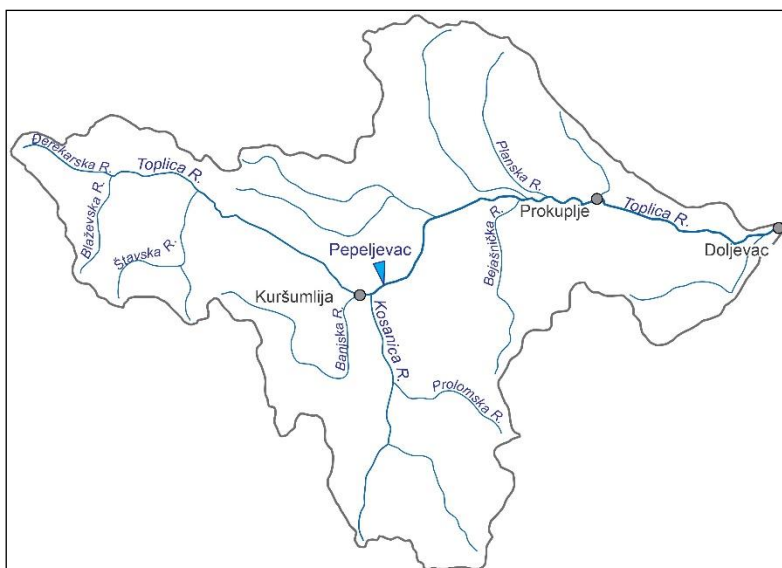


Fig. 1. The Toplica basin with hydrological profile Pepeljevac

One of the most important tasks in hydrological studies is an estimation of discharge and water level which can be expected in the future on a certain hydrological profile, based on the previous discharges. Discharge depends on a large number of factors and it is subject to laws of coincidence, which is why it can be studied by statistical methods. Statistical analysis of the probability of characteristic discharge occurrence on the rivers has been a subject of many hydrological studies (Gavrilović, 1988; Milanović, 2007; Urošev, 2007; Ristić, 2009; Dimitrijević, 2010; Đokić, 2010; Bolgov & Korobkina, 2013; Vasilevski & Radevski, 2014).

It is necessary to take as long data series as possible into account to obtain a reliable statistical analysis. To analyze average, high and low waters, the values of average annual, minimum and maximum annual discharges for the period from 1951 to 2014 from the hydrological station Pepeljevac were used. Hydrological station Pepeljevac is situated 69.5 km from the Toplica confluence into the Južna Morava. The surface of the river basin on the profile amounts to 986 km², with angle "0" elevation at 329.9 m altitude. The aim

of this paper is calculation of the probability of occurrence of average, minimum and maximum annual Toplica discharges, which is important for water management planning in the Toplica river-basin upstream the hydrological station Pepeljevac.

Study methods

The probability of occurrence of average, high and low waters in a river is done on the basis of the data on average, absolute maximum and minimum discharges, by maximal annual method. There is a great number of distribution functions used for calculating the probability of occurrence of low, high and average waters, but the most common in use is Pearson type III distribution (Gavrilović, 1988; Milanović, 2006, Đokić, 2015, Đokić et al., 2015; Dimitrijević et al., 2010; Kovačević-Majkić, 2009; Milijašević, 2010). To create the probability curve of average, maximum and minimum discharges of the Toplica, Pearson type III distribution was used.

On the basis of the probability of average annual discharge occurrence, a classification of years by water richness according to Ocokoljić (1994) was done. They were divided to: extremely dry, very dry, dry, moderately rich in water, rich in water, very rich in water and extremely rich in water.

Mann-Kendall test (Kendall, 1975) examined the trend of the Toplica discharge on the observed profile Pepeljevac in the measuring period from 1951 to 2014 (Martić Bursać, 2015). Homogeneity of the data was tested by Pettit, SNTH, Buishand and von Neumann tests (Radivojević et al., 2015).

Results and discussion

Statistical analysis of average waters

On the basis of the data on the Toplica River discharge in the period from 1951 to 2014, average monthly and annual discharge on the hydrological profile Pepeljevac were calculated (Fig. 2).

Maximum average monthly discharges in the analyzed period were recorded in April and March, while minimum average discharges occur at the end of summer and the beginning of autumn, that is, in August and September. The values of maximum average discharges in spring months are 48 – 49 % higher than average annual discharges. Minimum discharges at the end of summer and the beginning of autumn are a consequence of lower precipitation amount in the warmer part of the year, as well as high summer air temperatures and evaporation.

Homogeneity tests Pettit, SNTH, Buishand and von Neumann show that the data are homogenous, as well as that statistically important breakpoints in data series cannot be identified.

Mann-Kendall test determined that there is no statistically important trend of the Toplica River discharges on the observed profile Pepeljevac in the measuring period from 1951 to 2014.

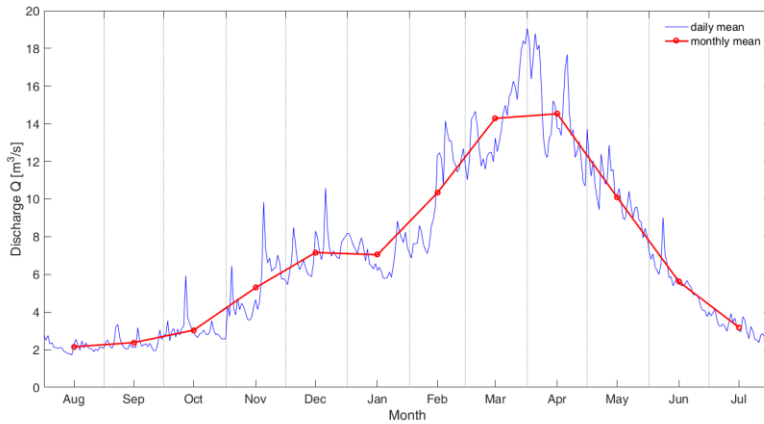


Fig. 2. Hydrograph of the Toplica River on the hydrological profile Pepeljevac in the period from 1951 to 2014

Classifying years by water richness

To classify years by water richness, a period from 1951 to 2014 was taken into account from the Pepeljevac station on the Toplica River. In the graph of average annual discharge values (Fig. 3) it can be noticed that certain years are very rich in water (1955 – 18.58 m³/s; 2006 – 13.3 m³/s; 1954 – 12.04 m³/s), while some others are scarce in water (1968 – 3.05 m³/s; 1994 – 3.32 m³/s; 1990 – 3.79 m³/s). Below average discharge is more present than above average discharge. During the study period of 64 years, the discharge was above average for 29 years, while it was under average for 35 years.

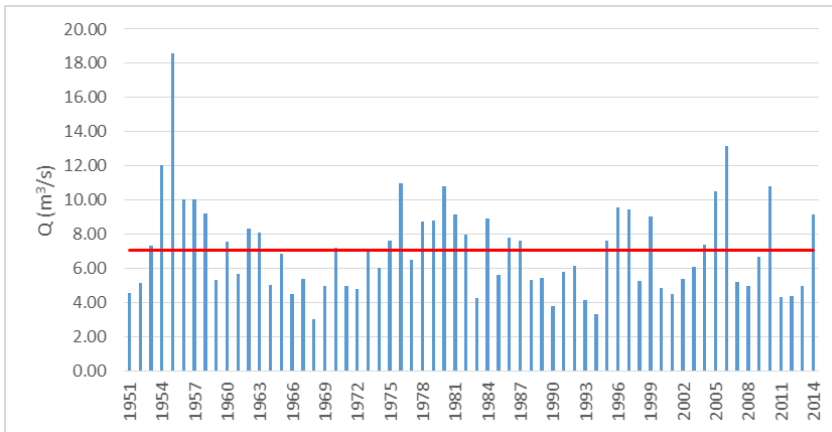


Fig. 3. Average annual discharge values of the Toplica in hydrological station Pepeljevac in the period from 1951 to 2014 and average annual discharge for the studied period

To forecast the Toplica River discharge we first calculated average multi-annual average discharge on the Pepeljevac station in the period from 1951 to 2014, which was 7.09 m³/s. Then we determined module coefficient (k) which helped in calculating coefficient of variation (C_v). This coefficient for the Toplica River is C_v = 0.38. It indicates

significant variability in average annual discharge. Coefficient of asymmetry (C_s) amounts to 1.40. Based on these parameters and Ribikin table, we calculated probabilities of average water level occurrence at the hydrological station Pepeljevac (Tab. 1) and they were shown graphically by probability curve of probability of average discharge occurrence (Fig. 4).

Tab. 1. Probability of average annual discharge occurrence on the Toplica River on the hydrological profile Pepeljevac in the period from 1951 to 2014

Probability	Return period	σ	$\sigma \cdot C_v$	$k_s = \sigma \cdot C_v + 1$	$Q_{av.}$
0.01	10000	6.87	2.62	3.62	25.68
0.1	1000	5.09	1.94	2.94	20.86
1	100	3.27	1.25	2.25	15.94
3	33.3	2.37	0.90	1.90	13.50
5	20	1.94	0.74	1.74	12.34
10	10	1.34	0.51	1.51	10.72
20	5	0.71	0.27	1.27	9.01
25	4	0.49	0.19	1.19	8.42
30	3.3	0.31	0.12	1.12	7.93
40	2.5	0.02	0.01	1.01	7.14
50	2	-0.22	-0.08	0.92	6.50
60	1.6	-0.44	-0.17	0.83	5.90
70	1.42	-0.64	-0.24	0.76	5.36
75	1.33	-0.73	-0.28	0.72	5.12
80	1.25	-0.83	-0.32	0.68	4.85
90	1.11	-1.04	-0.40	0.60	4.28
95	1.05	-1.17	-0.45	0.55	3.93
97	1.03	-1.23	-0.47	0.53	3.76
99	1.01	-1.32	-0.50	0.50	3.52
99.9	1	-1.39	-0.53	0.47	3.33

σ – deviation of the ordinate of the binominal asymmetric curve of provision (frequency) from the average (from 1.0) when $C_v=1.0$ (Ribikin table); C_v - coefficient of variation; k_s – module coefficient of the ordinate; $Q_{av.}$ – average discharge.

A classification of years by being rich in water (Ocokoljić, 1994) was done on the basis of Pearson type III distribution. According to this classification, the years were ranked by water richness for the Toplica in Pepeljevac in such a way: extremely dry (< 3.52 m³/s), very dry (3.52 – 3.93), dry (3.93 – 5.12), moderately rich in water (5.12 – 8.42), rich in water (8.42 – 12.34), very rich in water (12.34 – 15.94) and extremely rich in water (> 15.94) (Tab. 2).

In the studied period, the year of 1955 was the richest in water, which ranks it among years extremely rich in water, with the occurrence probability of such a year below 1 %. Average annual discharge amounted to 18.58 m³/s of water. Two years, 1968 (3.05 m³/s) and 1994 (3.32 m³/s) belong to the group of extremely dry years, with the probability of occurrence of such a year below 1 %. The highest number of years belongs to the group of years moderately rich in water, with occurrence probability of 25-70 %. One year is very dry year and the other is very rich in water. 1990 was very dry, when average annual discharge was 3.79 m³/s, while the one very rich in water was 2006, with 13.13 m³/s. There were somewhat more rich in water (16) than dry (14) years.

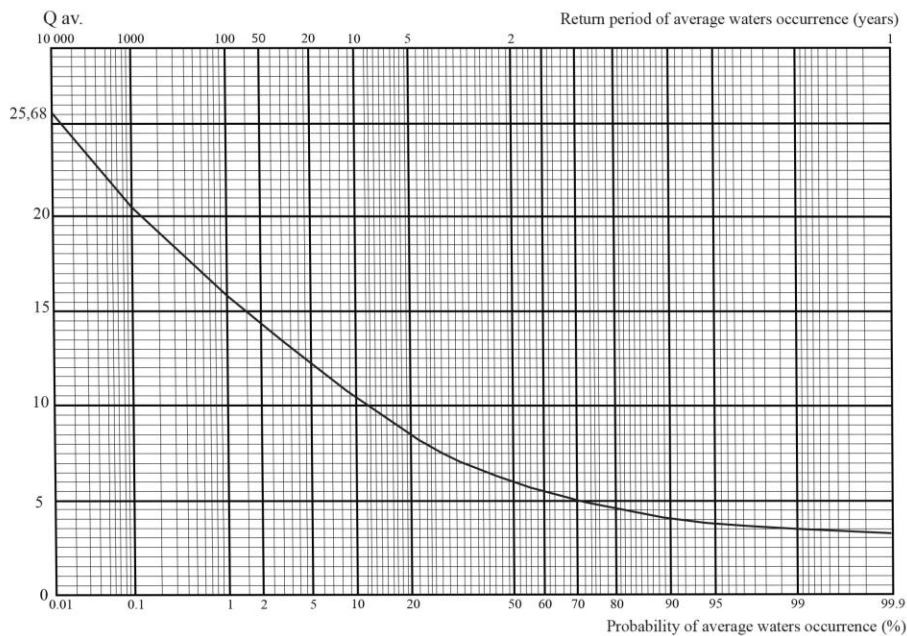


Fig. 4. Probability curve of average discharge occurrence in the Toplica River on the Pepeljevac profile

Tab. 2. Classifying years by water richness on the Toplica River in Pepeljevac in the period from 1951 to 2014

Years rich in water	Discharge (m ³ /s)	Years	Number of years
Extremely dry	<3.52	1968, 1994	2
Very dry	3.52–3.93	1990	1
Dry	3.93 – 5.12	1951, 1964, 1966, 1969, 1971, 1972 1983, 1993, 2000, 2001, 2008, 2011, 2012, 2013	14
Moderately rich in water	5.12 – 8.42	1952, 1953, 1959, 1960, 1961, 1962, 1963, 1965, 1967, 1970, 1973, 1974, 1975, 1977, 1982, 1985, 1986, 1987, 1988, 1989, 1991, 1992, 1995, 1998, 2002, 2003, 2004, 2007, 2009	29
Rich in water	8.42–12.34	1954, 1956, 1957, 1958, 1976, 1978, 1979, 1980, 1981, 1984, 1996, 1997, 1999, 2005, 2010, 2014	16
Very rich in water	12.34 – 15.94	2006	1
Extremely rich in water	>15.94	1955	1

The results obtained by forecasting the occurrence of average waters refer to the conclusion that the average discharge of 3.33 m³/s can be expected each year with 99.9 % occurrence probability, while the discharge of 3.52 m³/s can be expected with 99 % occurrence probability. Average annual discharge of 6.50 m³/s can be expected every other year. The lowest average annual discharge of 3.05 m³/s, which was recorded in 1968, can be expected each year.

Statistical analysis of high waters

In the period from 1951 to 2014, maximum waters occurred mostly in March and April. High waters are a consequence of abrupt snow melting in higher parts of the river-basin.

Absolute maximum discharge on the Pepeljevac station on the Toplica River in the observed period was recorded on 07 June 1976, and it was 468 m³/s. Absolute minimum discharge was recorded on 03 September 1952, and it was 0.085 m³/s. The ratio between absolute maximum and minimum discharges on the Toplica River in Pepeljevac during the analyzed period was 1:5506.

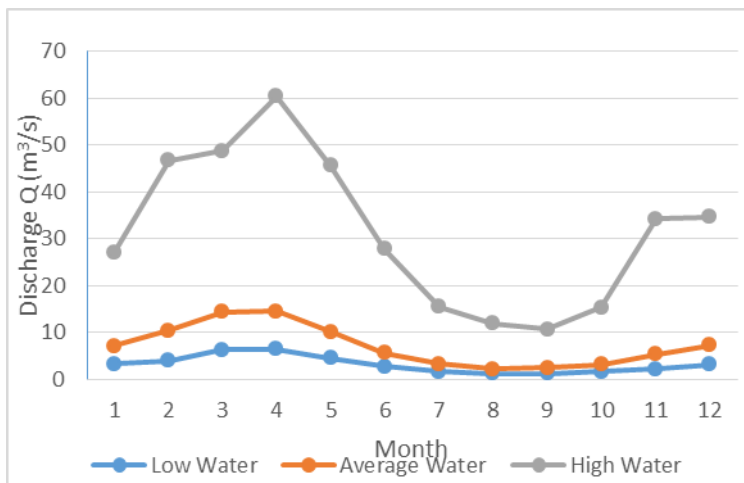


Fig. 5. High, average and low waters of the Toplica River on the hydrological profile Pepeljevac in the period from 1951 to 2014

To forecast maximum discharges of the Toplica River, we firstly calculated average multi-annual maximum discharge on the Pepeljevac station in the period from 1951 to 2014, which was 132.82 m³/s. Module coefficient (k) helped to calculate coefficient of variation, which is $C_v = 0.63$ for the Toplica River and points out to significant fluctuation in annual maximum discharges. Coefficient of asymmetry (C_s) is 1.79. On the basis of these parameters and Ribikin table the probabilities of maximum discharges on the hydrological station Pepeljevac were calculated (Tab. 3) and shown graphically by a curve of occurrence probability of maximum discharges (Fig. 6).

Tab. 3. Probable maximum discharges of the Toplica River on the Pepeljevac profile

Probability	Return period	σ	$\sigma \cdot C_v$	$k_s = \sigma \cdot C_v + 1$	Q_{max}
0.01	10000	7.76	4.86	5.86	777.74
0.1	1000	5.64	3.53	4.53	601.55
1	100	3.50	2.19	3.19	423.70
3	33.3	2.46	1.54	2.54	337.27
5	20	1.98	1.24	2.24	297.38
10	10	1.32	0.83	1.83	242.53
20	5	0.64	0.40	1.40	186.01
25	4	0.42	0.26	1.26	167.73
30	3.3	0.24	0.15	1.15	152.77
40	2.5	-0.05	-0.03	0.97	128.67
50	2	-0.28	-0.18	0.82	109.55
60	1.6	-0.48	-0.30	0.70	92.93
70	1.42	-0.64	-0.40	0.60	79.63
75	1.33	-0.72	-0.45	0.55	72.99
80	1.25	-0.80	-0.50	0.50	66.34
90	1.11	-0.94	-0.59	0.41	54.70
95	1.05	-1.02	-0.64	0.36	48.05
97	1.03	-1.06	-0.66	0.34	44.73
99	1.01	-1.09	-0.68	0.32	42.24
99.9	1	-1.10	-0.69	0.31	41.40

σ – deviation of the ordinate of the binominal asymmetric curve of provision (frequency) from the average (from 1.0) when $C_v=1.0$ (Ribikin table); C_v - coefficient of variation; k_s – module coefficient of the ordinate; Q_{max} – average maximum discharge.

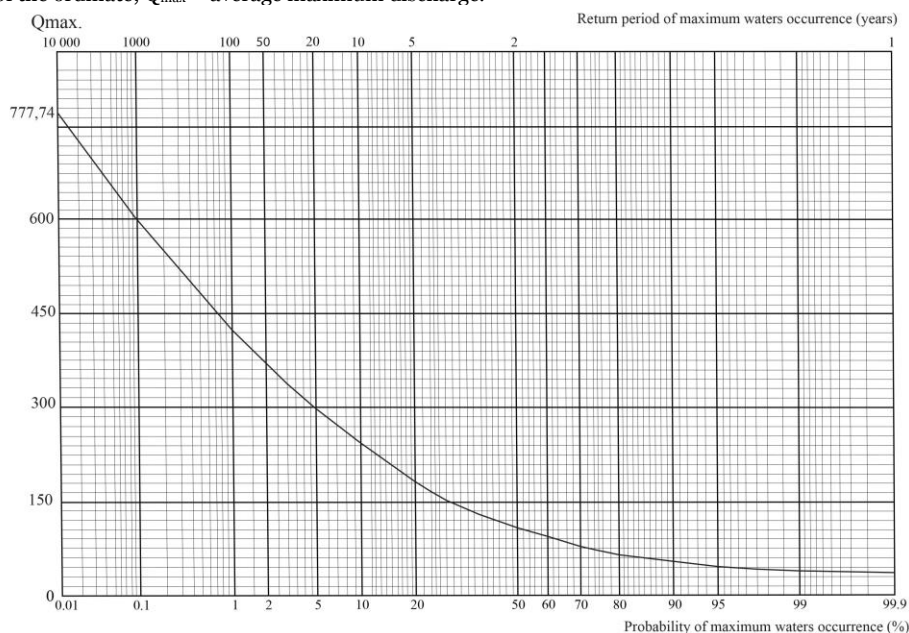


Fig. 6. Curve of occurrence probability of maximum discharges of the Toplica River on the Pepeljevac profile

The obtained results show that the maximum discharge of 41.40 m³/s can be expected each year with 99.9 % occurrence probability, while the discharge of 42.24 m³/s can be expected with 99 % probability. Maximum annual discharge of at least 109.55 m³/s can be expected every other year. Maximum recorded discharge of 468 m³/s, which was recorded in Pepeljevac in the period from 1951 to 2014, can be expected once in every 300 years.

Statistical analysis of low waters

In the summer period of a year, due to high air temperatures enhanced evaporation and evapotranspiration, the balance of surface waters is unfavorable. In the period from 1951 to 2014, minimum waters occurred mostly in August and September.

Low water prognosis was done by applying the same methods as with the high water prognosis. An average multi-annual minimum discharge on the Pepeljevac station in the period from 1951 to 2014 was calculated. It was 1.01 m³/s. This coefficient is $C_v = 0.55$ for the Toplica River. Coefficient of asymmetry (C_s) is 0.88. In order to avoid negative values it was calculated as $C_s = 2C_v$, ($0.55 \cdot 2 = 1.1$).

On the basis of these parameters and Ribikin table, the probabilities of minimum water levels occurrence on the hydrological station Pepeljevac were calculated (Tab. 4) and shown graphically by a curve of occurrence probability of minimum discharges (Fig. 7).

Tab. 4. Probable minimum discharges of the Toplica River on the hydrological profile Pepeljevac

Probability	Return period	θ	$\theta \cdot C_v$	$k_s = \theta \cdot C_v + 1$	Q_{min}
0.01	10000	6.18	3.41	4.41	4.46
0.1	1000	4.67	2.58	3.58	3.61
1	100	3.09	1.71	2.71	2.73
3	33.3	2.28	1.26	2.26	2.28
5	20	1.89	1.04	2.04	2.06
10	10	1.34	0.74	1.74	1.76
20	5	0.74	0.41	1.41	1.42
25	4	0.54	0.30	1.30	1.31
30	3.3	0.36	0.20	1.20	1.21
40	2.5	0.07	0.04	1.04	1.05
50	2	-0.18	-0.10	0.90	0.91
60	1.6	-0.41	-0.23	0.77	0.78
70	1.42	-0.62	-0.34	0.66	0.66
75	1.33	-0.74	-0.41	0.59	0.60
80	1.25	-0.85	-0.47	0.53	0.54
90	1.11	-1.10	-0.61	0.39	0.40
95	1.05	-1.28	-0.71	0.29	0.30
97	1.03	-1.38	-0.76	0.24	0.24
99	1.01	-1.52	-0.84	0.16	0.16
99.9	1	-1.68	-0.93	0.07	0.07

θ – deviation of the ordinate of the binominal asymmetric curve of provision (frequency) from the average (from 1.0) when $C_v=1.0$ (Ribikin table); C_v - coefficient of variation; k_s – module coefficient of the ordinate; Q_{min} – average minimum discharge.

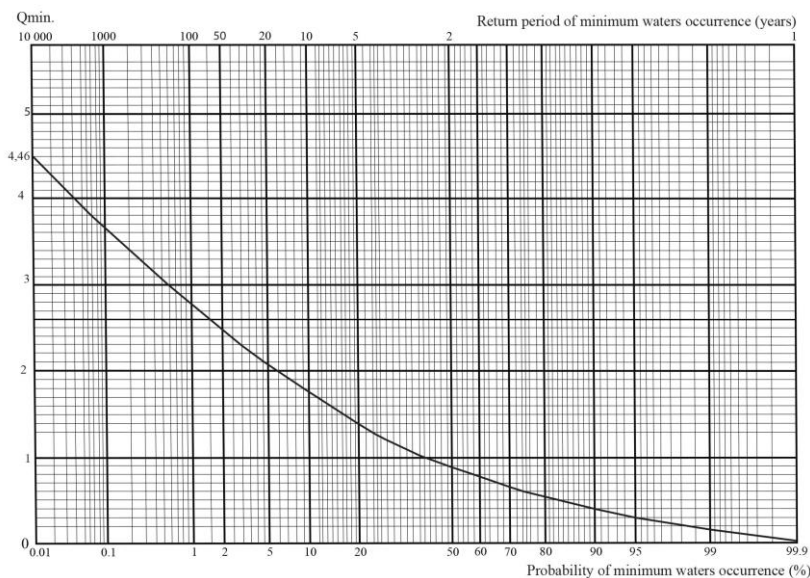


Fig. 7. Curve of occurrence probability of minimum discharges of the Toplica River on the hydrological profile Pepeljevac

The results obtained by this analysis refer to the conclusion that the minimum discharge of $0.07 \text{ m}^3/\text{s}$ can be expected once in 1000 years with 99.9 % non-overcoming probability, while the discharge of $0.16 \text{ m}^3/\text{s}$ can be expected with 99 % non-overcoming occurrence probability. The absolute recorded minimum discharge of $0.085 \text{ m}^3/\text{s}$, which was recorded in the analyzed period, can be expected every year. Low waters occurrence, the value of which is less than 1 m^3 occurs often, with probability of occurrence from 50 to 99.9 %.

Conclusion

On the basis of data on discharge in the period from 1951 to 2014 probability of average, minimum, maximum discharges occurrence on the Pepeljevac hydrological profile on the Toplica River was estimated.

Coefficient of variation for average annual discharges for the Toplica River is $C_v = 0.38$, which points to a significant oscillation in discharge. Maximum average monthly discharges in the analyzed period were recorded in April and March, while minimum average discharges occur at the end of summer and the beginning of autumn, that is, in August and September. Values of maximum average discharges in spring months are 48 – 49 % higher than average annual discharge. Minimum discharges at the end of summer and the beginning of autumn are a consequence of high summer temperatures and increased evaporation.

When the years were classified by water richness, during the 64-year analyzed period, discharge was above average value for 29 years, while it was below average for 35 years. There were somewhat more rich in water (16) than dry (14) years.

Coefficient of variation of maximum annual discharges for the Toplica River is $C_v = 0.63$, which points out to significant fluctuation in annual maximum discharges. Maximum recorded discharge of 468 m³/s, which was recorded in Pepeljevac in the period from 1951 to 2014, can be expected once in every 300 years.

Coefficient of variation of minimum annual discharges is $C_v = 0.55$, pointing out to exceptional fluctuation. By analyzing probability curve of minimum discharge occurrence, minimum discharge of 0.07 m³/s can be expected each year. Absolute minimum discharge of 0.085 m³/s, which was recorded in the analyzed period, can be expected every year. Such low waters regime can lead to significant problems in water management, especially during summer months.

Water management planning is significantly more aggravated because of exceptional fluctuation in discharge of the Toplica River. To abate the consequences of high and low waters, forming of reservoir Selova on the Toplica, the construction of which started in 1986, will be very important, as well as a series of other measures, such as defensive embankments, dams, watercourse regulation, deforestation, and so on.

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